Design of a Laboratory Assignment Institution without the Second Selection under the Sameness Assumption: Fairness, Strategy-Proof and Stability

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"Laboratory assignment" is a process to assign students to laboratories for their graduation research. It assumes the following conditions: every student should belong to one and only one laboratory, the maximum number of students for each laboratory is fixed and announced in advance, and the process of assignment depends only on the preferences of the students and teachers involved. Under this assumption, we consider a laboratory assignment institution for each student to be able to belong to some laboratory. An extended version of the matching institution which was proposed by Gale and Shapley (1962) not only satisfies the conditions required but also has some theoretically desirable properties. But this institution assumes, among others, that every student should, by preference, linearly order all the laboratories from which to choose only one for his/her graduation research, which, though theoretically valid, is not realistically plausible. To put it into practice, it should be examined whether assignment can be successful when the number is relatively small of the candidates a student should choose first from all the laboratories. Analysis by Monte Carlo simulation reveals that this institution works well even in such situations, given a certain set of conditions on the maximum number of students for each laboratory and the number of the candidates the students should submit first.

Key words and phrases: Laboratory Assignment, Sequential Selection Institution, Institution Design without Second Selection, Social Matching Theory, Monte Carlo Simulation

1. INTRODUCTION

"Laboratory assignment" is a process to assign students to laboratories for their graduation research. It assumes the following conditions: every student should belong to one and only one laboratory, the maximum number of students for each laboratory is fixed and announced in advance, and the process of assignment depends only on the preferences of the students and teachers involved. Under this assumption, we consider a laboratory assignment institution for each student to be able to belong to some laboratory.

Among the institutions ever proposed for the same or a similar purpose is "Sequential Selection Institution," which has actually been used in some faculties in Japanese universities. The main part of this institution is as follows. Some of all the students are assigned to somewhere laboratories at the first selection of laboratory assignment. Some of the other students who are unmatched at the first are assigned to somewhere laboratories at the second. Some of the other students who are unmatched at the second are assigned to somewhere laboratories at the third, and so on. This matching process is repeated until all students are assigned. This institution, however, has some problems: unfairness among students or laboratories in assignment processes, existence of incentives to show untrue preferences, and instability on assignment outcomes (Tomiyama and Hosono 1999).

Another institution is an extended version of the matching procedure that was proposed by Gale and Shapley (1962), using the social matching theory as a branch of the game theory. This institution not only satisfies the conditions required above but also avoids these problems (Roth 1982, 1984, 1985a, 1985b, 1985c, Roth and Sotomayer 1990, Tomiyama 1992). But this institution assumes, among others, that every student should, by preference, linearly order *all* the laboratories from which to choose only one for his/her graduation research, which, though theoretically valid, is not realistically plausible. To put it into practice, it should be examined whether assignment can be successful when the number is relatively small of the candidates a student should choose first from all the laboratories. According to Tomiyama and Hosono (1999), an analysis by Monte Carlo simulation reveals that this institution works well even in such situations, given a certain set of conditions on both the maximum number of students for each laboratory and the number of the laboratories the students should submit first.

The good result is obtained under several assumptions set up for the simulation. One of the assumptions is that the preference orderings of laboratories over the set of students are randomly generated from the uniform distribution in the computer program. Some faculties in Japanese universities, however, do not satisfy this *random assumption*. In their laboratory assignment institutions, the preference orderings of laboratories are the same. The sameness is actualized by use of a list of the students' records. Under this *sameness assumption*, it is not clear whether the above simulation result is preserved. The purpose of this paper is to investigate this situation by Monte Carlo simulation under the same assumptions in Tomiyama and Hosono (1999), except for the sameness assumption.

2. A MONTE CARLO SIMULATION: METHOD AND PARAMETERS

The matching procedure that was proposed by Gale and Shapley is outlined in the following. It has the six parameters:

NOS	the number of students
NOL	the number of laboratories
MNS	the maximum number of students who can belong to each laboratory
RPO	the ranking of preference ordering which a student submits first
POS	the preference ordering of a student over the set of laboratories

POL the preference ordering of a laboratory over the set of students

Given a set of the first four parameters, NOS, NOL, MNS and RPO, this procedure makes a matching outcome between students and laboratories by use of only the information about a certain set of POSs and POLs. On this matching outcome, each student may be matched to some laboratory, or some of the students may be matched and the others unmatched. (For more information about the procedure, see Gale and Shapley (1962) or Tomiyama (1992).)

Whether each student can be matched to some laboratory depends on the values of the parameters. In order to suitably determine the values used in the simulation, we will have to consider about their characteristics in a real laboratory assignment. At any time of laboratory assignment, it is considered that the values of NOS and NOL are exogenously determined and fixed. This means that their values are uncontrollable to the designer. But they may be different at the first of the academic year. Since POLs are arbitrary fixed because of the sameness assumption, they also are uncontrollable. POSs are also uncontrollable, but are freely and randomly revealed by the students. On the other hand, the values of MNS and RPO are normally considered to be controllable within certain constraints.

Judging from the above, we determined the range of parameters’ values and several assumptions used in the simulation as follows. Table 1 shows the range of the first four parameters. It is assumed through the simulation that all MNSs are the same among the laboratories and all RPOs are the same among the students. POSs are randomly and independently generated from the uniform distribution in the computer program. But a certain POL is fixed in it as the same among the laboratories. In the

Table 1. Range of parameters: Initial, final and incremental values

Parameters	Initial	Final	Incremental
NOS	20	200	20
NOL	10	50	5
MNS	the minimum integer more than NOS/NOL	20（NOS=20,40,60）	1
		25（NOS=80,100）	
		30（NOS=120,140）	
		35（NOS=160,180,200）	

simulation, the average number of unmatched students for 10,000 sets of POSs is calculated under the sameness assumption about POL, given every set of the values of NOS, NOL, MNS and RPO in Table 1. We call this average the "*u*-number" below.

3. ANALYSIS OF SIMULATION RESULTS

3.1 MNS and RPO Making the u-Number Zero

Analysis of the u-number for a pair of NOS and NOL. As mentioned in the preceding section, it is considered that a pair of NOS and NOL is fixed at any time of laboratory assignment. So, first, we shall analyze the *u*-numbers for one pair and investigate their characteristics.

Table 2 shows the *u*-numbers for all the pairs of MNS and RPO in Table 1 when NOS is 120 and NOL is 35. All of the *u*-numbers in the table are rounded off to one decimal place. This is same in the other cases. This table tells us that the two outcomes hold:

Outcome 1: For any MNS, the *u*-number decreases monotonically as RPO increases.
Outcome 2: For any RPO, the *u*-number decreases monotonically as MNS increases.

These outcomes are identical to our intuition on the relation between MNS and RPO.

Let us introduce here some terminology for later discussions to be convenient. Assume that a pair of NOS and NOL is fixed. We will use the term the “zero-RPO” to refer to the

Table 2. The *u*- numbers (NOS=120, NOL=35)

	RPO										
MNS	1	2	3	4	5	6	7	8	9	10	～	35
4	16.8	6.8	3.1	1.5	0.8	0.4	0.2	0.1	0.1	0		0
5	7.7	1.4	0.3	0.1	0	0	0	0	0	0		0
6	3.2	0.3	0	0	0	0	0	0	0	0		0
7	1.2	0	0	0	0	0	0	0	0	0	～	0
8	0.4	0	0	0	0	0	0	0	0	0		0
9	0.1	0	0	0	0	0	0	0	0	0		0
10	0	0	0	0	0	0	0	0	0	0		0
～						～					～	
30	0	0	0	0	0	0	0	0	0	0		0

minimum RPO making the *u*-number zero for a MNS and the “zero-MNS” to the minimum MNS making the *u*-number zero for a RPO. The term “zero-line” can be defines as the set of pairs of MNS and RPO satisfying *both* the zero-MNS and the zero-RPO. In Table 2, for example, the zero-RPO for MNS 4 is 10, the zero-MNS for RPO 1 is 10, and the zero-line is consisted of all 0s.

Outcome 1 implies that there is only one zero-RPO for a MNS, and Outcome 2 implies that there is only one zero-MNS for a RPO. Therefore, it is implied that there is only one zero-line for a given pair of NOS and NOL. Furthermore, the two outcomes imply the followings.

Outcome 3: When MNS is minimal and RPO is 1, the *u*-number is maximal.
Outcome 4: When MNS is minimal, the zero-RPO is maximal.
Outcome 5: When RPO is 1, the zero-MNS is maximal.
Outcome 6: The zero-RPO decreases monotonically as MNS increases.
Outcome 7: The zero-MNS decreases monotonically as RPO increases.

*Analysis of the u-number for all pairs of NOS and NOL***.** Outcomes 1 and 2 satisfied for a pair of NOS 120 and NOL 35 were satisfied for *all* pairs of NOS and NOL in Table 1. All of the zero-lines under the sameness assumption are shown in Table 3.

According to Tomiyama and Hosono (1999), the outcomes also held under the randomness assumption. All of the zero-lines under this assumption are shown in Table 4. The table will be used in the last section when the zero-lines under the two assumptions are compared with. Before turning to there, we must draw attention to some implications of the analyzed result to the design of a laboratory assignment institution without the second selection.

3.2 Implications to the Design of an Institution without the Second Selection

A laboratory assignment institution making the *u*-number zero, that is an institution without the second selection, can be designed by selecting a pair of MNS and RPO in Table 3 under a given pair of NOS and NOL. There are several pairs of MNS and RPO on the zero-line. Which pair should the designer select from them? In order to get some suggestion to answer this question, we have to inquire into two things; one is about characteristics of the zero-line and the other is about requirements (or constraints) for MNS and/or RPO in a real laboratory assignment. Let us consider them in order.

The zero-line has such a basic characteristic that the smaller MNS is, the larger RPO is. As MNS is smaller, the numbers of the students assigned to laboratories are more equalized

On the side of laboratories,

smaller	←	MNS	→	larger
more	←	Equalization	→	less

On the side of students,

larger	←	RPO	→	smaller
wider	←	Difference	→	narrower
heavier	←	Burden	→	lighter

Figure 1. The characteristics of the zero-line

among them. As RPO is larger, on the other hand, the difference among the students is wider in the sense that their rankings for laboratories in a matching outcome disperse. In addition, the burden on the side of students is heavier in the sense that they must linearly rank a number of laboratories by their preferences. Figure 1 summaries these relationships. It seems that the equalization, differences and burden are useful as criteria in a real situation of laboratory assignment for the designer to select a pair of MNS and RPO from several pairs on the zero-line.

In a real laboratory assignment, there may or not be some requirements for MNS and/or RPO. This takes part in whether the designer has to select MNS and RPO simultaneously as a pair or sequentially in some order. We shall discuss it in detail. To begin with, assume that there is no requirement for them. To make the *u*-number zero then, it is desirable to select some MNS and some RPO on the zero-line simultaneously. The reason is that any pair on the zero-line is *the most efficient* of all possible pairs in the sense that RPO is always minimal to make the *u*-number zero for a given MNS, and vice versa. Next, assume that there is some requirement for MNS and/or RPO. A typical example of such a requirement may be that some constraint is given to the MNS from viewpoint of educational effects. In this case, some particular MNS is determined first as the MNS adopted in the laboratory assignment institution. In order to make the *u*-number zero under the institutional MNS, the designer has to select the zero-RPO for it. Such zero-RPO can be easily calculated on the zero-line in Table 3. We should notice that this zero-RPO is not necessarily on the zero-line. In Table 2, for example, if the institutional MNS is 8, then the zero-RPO for it is 2. This pair of MNS and RPO is not on the zero-line.

4. DISCUSSION AND CONCLUSIONS

In the previous section, we discussed how to make a decision about the selection of MNS and RPO when a pair of NOS and NOL was given and fixed. In this final section, we shall discuss some characteristics of the MNS and RPO selection regardless of NOS and NOL, comparing the simulation results under the sameness assumption in Table 3 with them under the randomness assumption in Table 4.

Analyzing the simulation results in Table 4, Tomiyama and Hosono (1999) made a summary of the characteristics under the randomness assumption (R) in the following.

(R1) There is a pair of zero-MNS and zero-RPO on the zero-line only when MNS is the minimum (i.e., k=0) and the minimum plus 1 (i.e., k=1).

(R2) When MNS is the minimum, the minimum, maximum and average of RPO are 4, 24 and 8.5 respectively. Since they show that RPO is large, the minimum MNS is not appropriate to use in a real design of laboratory assignment institution.

(R3) When MNS is the minimum plus 1, the minimum, maximum and average of RPO are 2, 5 and 3.7 respectively. Since they show that RPO is small, the minimum plus 1 of MNS is reasonable to do in it.

Analyzing the simulation results in Table 3 in a similar manner, we can summarize the characteristics of the MNS and RPO selection regardless of NOS and NOL under the sameness assumption (S) as follows.

(S1) There is a pair of zero-MNS and zero-RPO on the zero-line only when MNS is the minimum (i.e., k=0) and the minimum plus 1 (i.e., k=1).

(S2) When MNS is the minimum, the minimum, maximum and average of RPO are 5, 49 and 15.6 respectively. Since they show that RPO is large, the minimum MNS is not appropriate to use in a real design of laboratory assignment institution.

(S3) When MNS is the minimum plus 1, the minimum, maximum and average of RPO are 2, 8 and 5.2 respectively. Since they show that RPO is a little large, the minimum plus 1 of MNS may not be appropriate to do in it.

(S4) When MNS is the minimum plus 2, there is a pair of zero-MNS and zero-RPO on the zero-line, except that NOS is 20 and NOL is 35 or 40. But these two cases are not problematic because NOS is larger than NOL for almost every situation in real. MNS being the minimum plus 2, the minimum, maximum and average of RPO are 1, 5 and 3.4 respectively. Since they show that RPO is small, the minimum plus 2 of

MNS is reasonable to do in it.

From the above discussions, we conclude that the matching procedure proposed by Gale and Shapley is applicable enough regardless of NOS and NOL when MNS is the minimum plus 1 under the randomness assumption and when MNS is the minimum plus 2 under the sameness assumption.

It should be noticed that this conclusion holds under the condition that the preference orderings of students over the set of laboratories are randomly generated from the uniform distribution. In real situations of laboratory assignment, however, this condition is not necessarily satisfied. For example, we may observe social phenomena that there are several groups of students and all the members in every group have some identical preference ordering. It is not clear whether the conclusion hold even under this situation. This is open to be solved.

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